$\frac{39}{40} = A+$ $\cos\left(\frac{\pi}{3}\right)$ $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

AP Calculus BC Test

Part A. **Multiple Choice.** NO Calculator. Circle the letter of the best answer. Name Jake Choi

1. If $x = 3 + t^2$ and $y = \sin(2t)$, find dy/dx at $t = \pi/6$

$\frac{dy}{dx} = \frac{2\cos(2t)}{2t} = \frac{\cos(2t)}{t} = \frac{\frac{1}{2}}{\frac{\pi}{6}} = \frac{6}{2\pi} = \frac{3}{\pi}$

(A) $\frac{(3)^{1/2}}{3 + \pi^2/36}$ (B) $\pi/3$ (C) $3/\pi$ (D) $\frac{1/2}{3 + \pi^2/36}$

2. If $r^2 = 4\cos^2(4\theta)$, find $dr/d\theta$ at $(2, \pi/12)$

$2r \cdot \frac{dr}{d\theta} = 8 \cdot \cos(4\theta) \cdot (-4\sin(4\theta))$

$4 \cdot \frac{dr}{d\theta} = -32\cos(4\theta)\cdot\sin(4\theta)$ $-8 \cdot \left(\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} = -2\sqrt{3}$

(A) $-4\,(3)^{1/2}$ (B) $-2\,(3)^{1/2}$ (C) $2\,(3)^{1/2}$ (D) $4\,(3)^{1/2}$

3. A particle moves in the x-y plane so that its position for $t > 0$ is given by the equations $x = \ln(t + 1)$ and $y = kt^2$., where k is a positive constant. The line tangent to the particle's path at the point where $t = 3$ has slope = 8. What is the value of k?

$\frac{dy}{dx}\Big|_{t=3} = \frac{2kt}{\frac{1}{t+1}} = 2kt(t+1) = 8 \quad t=3$

$6k \times 4 = 8 \quad 24k = 8 \quad k = \frac{1}{3}$

(A) 1/192 (B) 1/3 (C) 4/3 (D)16/3

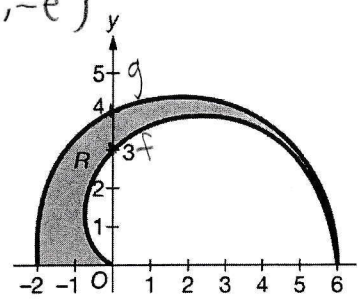
4. The position of a particle is given by the parametric equations $x(t) = \ln(t^2 + 1)$ and $y(t) = e^{3-t}$. What is the velocity vector at time $t = 1$?

$\frac{dx}{dt} = \frac{2t}{t^2+1} = \frac{2}{2} = 1 \quad \frac{dy}{dt} = -e^{3-t} = -e^2$

$(1, -e^2)$

(A) $(1, e^2)$ (B) $(1, -e^2)$ (C) $(1/2, e^2)$ (D) $(1/2, -e^2)$

5. Let **R** be the region in the first and second quadrants between the graphs of the polar curves $f(\theta) = 3 + 3\cos(\theta)$ and $g(\theta) = 4 + 2\cos(\theta)$, as shown in the figure at the right. Which of the following integral expressions gives the area of R?



(A) $\int_{-2}^{6} (g(\theta) - f(\theta))\,d\theta$

(B) $\int_{0}^{\pi} (g(\theta) - f(\theta))\,d\theta$

(C) $\frac{1}{2}\int_{0}^{\pi} (g(\theta) - f(\theta))^2\,d\theta$

(D) $\frac{1}{2}\int_{0}^{\pi} \left((g(\theta))^2 - (f(\theta))^2\right)\,d\theta$

$\frac{1}{2}\int_{0}^{\pi} \left(g(\theta)^2 - f(\theta)^2\right) d\theta$

$5 - 2t = 3 \rightarrow t = 1$

$t^2 - 3 = -2 \quad t^2 = 1 \rightarrow t = 1$

$(x'(t), y'(t)) = (-2, 2t) \quad (-2, 2)$

6. A particle moves in the x-y plane with position given by $(x(t), y(t)) = (5-2t, t^2 - 3)$ at time t. In which direction is the particle moving as it passes through the point (3, -2)

$t = 1$

(A) Up and to the left (B) Down and to the left (C) Up and to the right (D) Down and to the right.

7. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

$y = r\sin\theta = (1 + 2\sin\theta)\cdot\sin\theta$

$x = r\cos\theta = (1 + 2\sin\theta)\cos\theta \quad \theta = 0$

$\frac{dy}{dx} = \frac{2\cos\theta\sin\theta + (1+2\sin\theta)\cdot\cos\theta}{2\cos^2\theta - (1+2\sin\theta)\sin\theta} = \frac{\cos\theta}{2\cos^2\theta} = \frac{1}{2\cos\theta} = \frac{1}{2}$

(A) 2 (B) 1/2 (C) 0 (D) -1/2

AP Calculus BC Test (5)

Part A. **Multiple Choice.** **NO** Calculator Circle the letter of the best answer. Name Jake Choi

1. $\int_1^e x^4 \ln x \, dx =$

(A) $\frac{6e^5 - 1}{25}$ ✓ (B) $\frac{4e^5 + 1}{25}$ (C) $\frac{1 - e^3}{3}$ (D) e^4

$u = \ln x \quad v = \frac{x^5}{5}$

$u' = \frac{1}{x}\,dx \quad v' = x^4\,dx$

$\frac{x^5}{5}\ln x - \int_1^e \frac{x^4}{5}\,dx$

$= \frac{x^5}{5}\ln x - \frac{1}{25}x^5 \Big]_1^e$

$= \left(\frac{e^5}{5} - \frac{e^5}{25} + \frac{1}{25}\right)$

$= \frac{4e^5 + 1}{25}$

2. $\int \frac{8x - 10}{(2x-1)(x+1)}\,dx =$

(A) $-4\ln|2x-1| + 6\ln|x+1| + C$

✓ (B) $-2\ln|2x-1| + 6\ln|x+1| + C$

(C) $3\ln|2x-1| - 4\ln|x+1| + C$

(D) $6\ln|2x-1| - 4\ln|x+1| + C$

$\frac{A}{2x-1} + \frac{B}{x+1}$

$(A+2B)x + A - B = 8x - 10$

$A + 2B = 8$

$A - B = -10$

$3B = 18$

$B = 6$

$A = -4$

$-2\ln|2x-1| + 6\ln|x+1| + C$

3. $\int_0^{\ln 2} \frac{e^x}{1 + (e^x - 1)^2}\,dx =$

✓

(A) $\arctan(\ln 2)$ (B) $\ln 2$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

$e^x - 1 = u \quad e^x\,dx = du$

$\int \frac{1}{1+u^2}\,du$

$\arctan(u)$

$\arctan(e^x - 1)\Big]_0^{\ln 2}$

$= \arctan(1) - \arctan(0)$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

4. $\lim_{x \to 3} \frac{\tan(x-3)}{3e^{x-3} - x}$ is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) nonexistent

$= \frac{\sec^2(x-3)}{3e^{x-3} - 1} = \frac{1}{3-1} = \frac{1}{2}$ ✓

5 The function N satisfies the logistic differential equation $\frac{dN}{dt} = \frac{N}{10}\left(1 - \frac{N}{850}\right)$, where $N(0) = 105$. Which of the following statements is false?

(A) $\lim_{t \to \infty} N(t) = 850$ ✓

(B) $\frac{dN}{dt}$ has a maximum value when $N = 105$.

(C) $\frac{d^2N}{dt^2} = 0$ when $N = 425$. inflection

(D) When $N > 425$, $\frac{dN}{dt} > 0$ and $\frac{d^2N}{dt^2} < 0$. ✓

$\frac{850}{2} = 425$ max

425

(5)

$\frac{8}{8} = A$

AP Calculus BC Quiz

Name Jake Choi

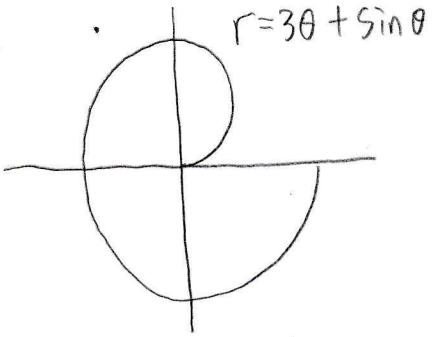
Part A. **Free Response.** Calculator Active.

1. The polar curve **r** is given by $\mathbf{r = 3\theta + \sin\theta}$ for $0 \leq \theta \leq 2\pi$.

(a) Find the area in the **second quadrant** enclosed by the graph of r and the coordinate axes. (3 points)

(b) Write an expression for dy/dθ and evaluate dy/dθ at θ = 2π/3. (3 points)

(c) A particle is traveling along the polar curve r so that its position (x(t), y(t)) is such that dθ/dt = 2 Find dy/dt at the instant that θ =2π/3, and interpret the meaning of your answer in the context of the problem. (2 points)



(a)
$$A = \frac{1}{2}\int_{\pi/2}^{\pi} (3\theta+\sin\theta)^2 \, d\theta = \boxed{47.513}$$ 3/3

(b)
$$y = r\sin\theta = (3\theta+\sin\theta)\cdot\sin\theta$$

$$\frac{dy}{d\theta} = \left((3+\cos\theta)\cdot\sin\theta + (3\theta+\sin\theta)\cdot\cos\theta\right) \qquad \theta = \frac{2\pi}{3}$$

$$= \left(\frac{5}{2}\cdot\frac{\sqrt{3}}{2} + \left(2\pi+\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{1}{2}\right)\right) = \boxed{-1.410}$$ 3/3

(c)
$$\frac{dy}{dt} = (-1.409542)\cdot\frac{d\theta}{dt} \qquad \frac{d\theta}{dt} = 2$$ 2/2

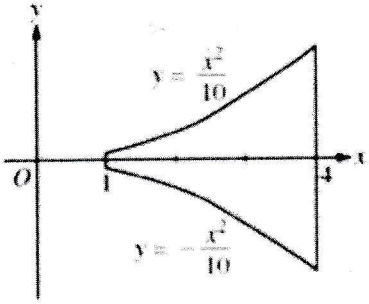
$$\frac{dy}{dt} = \boxed{-2.819}$$

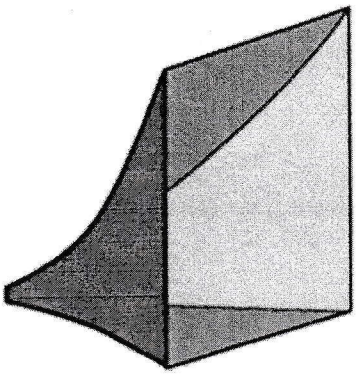
The particle is moving closer to the origin at $\theta = \frac{2\pi}{3}$ since $\frac{dy}{dt} < 0$ when $\theta = \frac{2\pi}{3}$, meaning that the radius of the polar curve is decreasing.

21/22 = A

AP Calculus BC Test

Part A. **Multiple Choice.** Calculator Active. Circle the letter of the best answer. Name Jake Choi





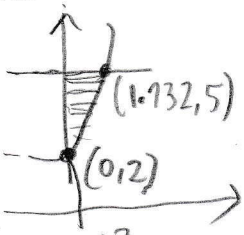
1. The base of a loudspeaker is determined by the two curves $y = \frac{x^2}{10}$ and $y = -\frac{x^2}{10}$ for $1 \le x \le 4$, as shown in the figure above. For this loudspeaker, the cross sections perpendicular to the x-axis are squares. What is the volume of the loudspeaker, in cubic units?

(A) 2.046 (B) 4.092 (C) 4.200 (D) 8.184 (E) 25.711

$\int_1^4 \left(\frac{x^2}{5}\right)^2 dx = 8.184$

2. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464 (B) 4.500 (C) 7.854 (D) 10.883 (E) 14.137



$V = \pi \int_2^5 \left(\sqrt{y-2}\right)^2 \cdot dy$

3. Let R be the region bounded by the graphs of $y = e^x$, $y = e^3$, and $x = 0$. Which of the following gives the volume of the solid formed by revolving R about the line $y = -1$?

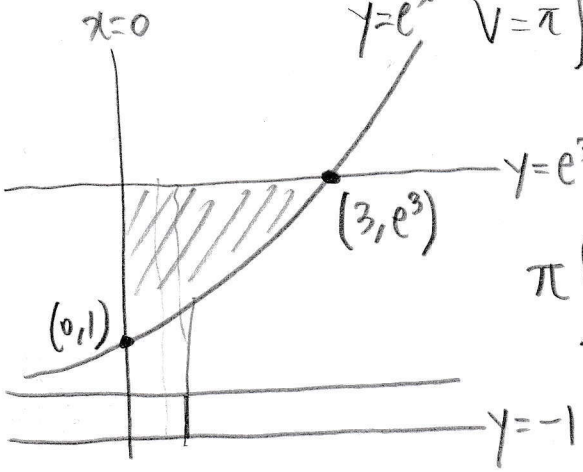
(A) $\pi \int_0^3 \left(e^3 - e^x + 1\right)^2 dx$

(B) $\pi \int_0^3 \left(e^3 - e^x - 1\right)^2 dx$

(C) $\pi \int_0^3 \left[\left(e^3 - e^x\right)^2 + 1\right] dx$

(D) $\pi \int_0^3 \left[\left(e^3 - e^x\right)^2 - 1\right] dx$

(E) $\pi \int_0^3 \left[\left(e^3 + 1\right)^2 - \left(e^x + 1\right)^2\right] dx$

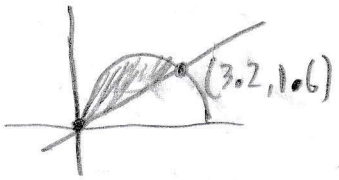


$\pi \int_0^3 \left((1+e^3)^2 - (1+e^x)^2\right) dx$

4. What is the area of the region enclosed by the graphs of $y = \sqrt{4x - x^2}$ and $y = \frac{x}{2}$?

(A) 1.707 (B) 2.829 (C) 5.389 (D) 8.886 (E) 21.447

(4) $\int_0^{3.2} \left(\sqrt{4x - x^2} - \frac{x}{2}\right) dx$



GOOD WORK!

50/52 = 91

AP Calculus BC Test

Part A. **Multiple Choice. NO** Calculator. Circle the letter of the best answer. Name Jake Choi

1. Let F be a function defined for all real numbers x such that $F'(x) > 0$ and $F''(x) > 0$. Which of the following could be a table of values for F?

in Con up

(A)

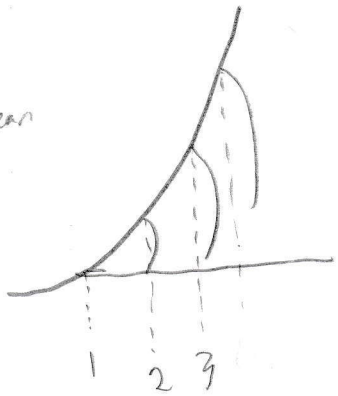
x	$F(x)$
1	−3
2	−4
3	−6
4	−9

X

(C)

x	$F(x)$
1	−3
2	0
3	3
4	6

X linear

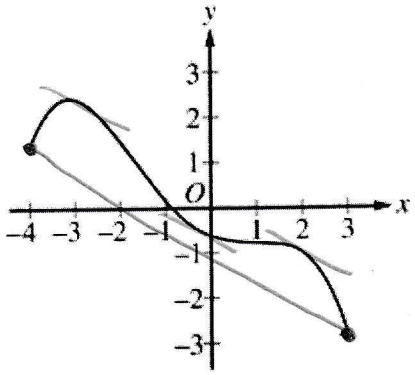


(B)

x	$F(x)$	
1	−3	
2	−1	+2
3	3	+4
4	19	+16

(D)

x	$F(x)$	
1	−3	
2	5	+8
3	11	+6
4	13	+2



Graph of f

2. The graph of a differentiable function f is shown above on the closed interval $[-4, 3]$. How many values of x in the open interval $(-4, 3)$ satisfy the conclusion of the Mean Value Theorem for f on $[-4, 3]$?

(A) Zero (B) One (C) Two (D) Three

3. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

$f'(x) = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

+ − +

−1 2

$-1 < x < 2$

(3)

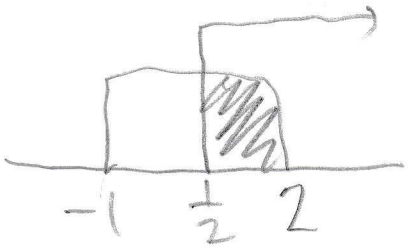
$f''(x) = 12x - 6 = 0$

$x = \frac{1}{2}$

− +

$\frac{1}{2}$

$x > \frac{1}{2}$



43/44 = A+

AP Calculus BC Test (1)

Part A. **No Calculator** Multiple Choice. Name Jake Choi

1. If $y^2 - 2x^2y = 8$, then $\frac{dy}{dx} =$

(A) $\frac{4}{y - 2x}$ (B) $\frac{2xy}{y - x^2}$ (C) $\frac{4 + 2xy}{y - x^2}$ (D) $\frac{2xy}{y + x^2}$ (E) $\frac{2xy + x^2}{y}$

2. If $f(x) = \sin x + 2x + 1$ and g is the inverse function of f, what is the value of $g'(1)$?

(A) $\frac{1}{3}$ (B) 1 (C) 3 (D) $\frac{1}{2 + \cos 1}$ (E) $2 + \cos 1$

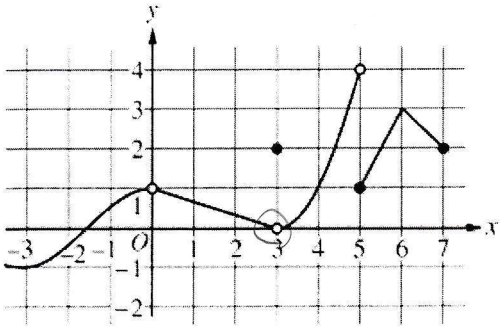
3. If $e^x - y = xy^3 + e^2 - 18$, what is the value of $\frac{dy}{dx}$ at the point $(2, 2)$?

(A) $e^2 - 32$ (B) $\frac{e^2 - 9}{24}$ (C) $\frac{e^2 - 8}{25}$ (D) $\frac{e^2}{13}$

4. The volume of a sphere is increasing at a rate of 6π cubic centimeters per hour. At what rate, in centimeters per hour, is its diameter increasing with respect to time at the instant the radius of the sphere is 3 centimeters? (Note: The volume of a sphere with radius r is given by $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{1}{3}$ (B) 1 (C) $\sqrt{6}$ (D) 6

5.



Graph of f

The graph of the function f is shown above. Which of the following statements is true?

(A) f is discontinuous at $x = 0$ because $\lim_{x \to 0} f(x)$ does not exist.

(B) f is discontinuous at $x = 3$ because $\lim_{x \to 3} f(x) \neq f(3)$.

(C) f is discontinuous at $x = 5$ because $\lim_{x \to 5^-} f(x)$ does not exist.

(D) f is discontinuous at $x = 6$ because $\lim_{x \to 6^-} f'(x) \neq \lim_{x \to 6^+} f'(x)$.

(5)

39/39 = A+

AP Calculus BC Test (6)

Part A. **NO Calculator**. Show all necessary steps. Name Jake Choi

1.Determine the following limits. If a limit does not exist, justify your reasoning.

(a) $\lim_{x \to 0} \frac{(x+4)^{1/2} - 2}{x}$ (b) $\lim_{x \to 2^-} f(x)$ if $f(x) = \begin{cases} \ln x \text{ for } 0 < x < 2 \\ x^2 \ln 2 \text{ for } 2 \leq x \leq 4 \end{cases}$ (c) $\lim_{t \to 0} \frac{(1 - \cos t)^2}{t}$

(d) $\lim_{x \to \text{negative infinity}} \frac{(3 + 2^x)}{(4 - 5^x)}$ (e) $\lim_{x \to 0} \frac{5x^5 + 3x^2 + 18x}{3x^5 + 6x}$

2. Determine the following one-sided limits

(a) $\lim_{x \to 0^-} \frac{(x+2)}{\cot x}$ (b) $\lim_{x \to -4^-} \left(x^2 + \frac{2}{x+4}\right)$ (c) $\lim_{x \to \pi/2^-} \frac{\tan x}{\sin x}$

3.

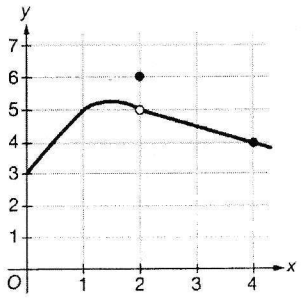
x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)	7.018	7.007	7.002	6.998	5.982	5.887

The table above gives selected values for a continuous function f. Based on the data in the table, what is the best approximation for $\lim_{x \to 4} f(x)$

4. Let f be the piecewise function defined by:

$$f(x) = \begin{cases} -x^2 + 3x + 3 \text{ for } x < 2 \\ 6 \text{ for } x = 2 \\ 6 - x/2 \text{ for } x > 2 \end{cases}$$

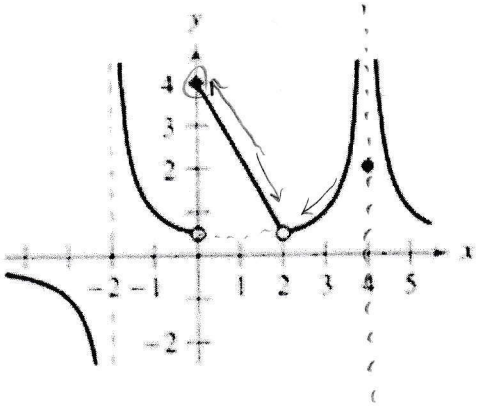
Part of the graph of f is shown at the right.
What is the value of $\lim_{x \to 2} f(f(x))$



5. Let f be the graph shown at the right. Determine the following limits. If a limit does not exist state your reasoning using correct mathematical terms.

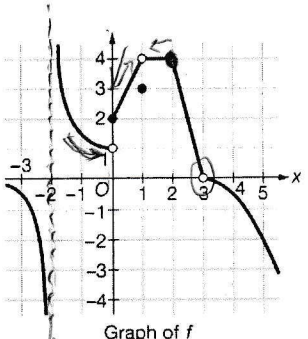
(a) $\lim_{x \to 2} f(x)$ (b) $\lim_{x \to 0^+} f(x)$

(c)Explain, using mathematical terms why f has a discontinuity at x = 0



6.

f(x)



Graph of f

Values of $g(x)$

x	0.8	0.9	0.99	0.999	1.001	1.01	1.1	1.2
$g(x)$	4.16	4.59	4.960	4.996	5.004	5.040	5.39	5.76

5