AP Calculus BC Final Assignment

Selected Assignment: More Calculus (#4) Given State: Northern Territory of Australia

(a) Estimate the area using a trapezoidal sum with 10 equal subdivisions



1cm = 62.5 km (Map Scale)

Table 1 (cm)

x (cm)	0	1.41	2.82	4.23	5.64	7.05	8.46	9.87	11.28	12.69	14.1
y (cm)	19.3	21.5	22.9	23.4	24.8	24.2	23.8	23.4	23.1	23.4	16.9

Table 2 (km)

x (km)	0	88.125	176.25	264.375	352.5	440.625	528.75	616.875	705	793.125	881.25
y (km)	1206.25	1343.75	1431.25	1462.5	1550	1512.5	1487.5	1462.5	1443.75	1462.5	1056.25

Calculation of the Riemann Sum (+Islands)

$$Area = \frac{1}{2} \times 88.125 \times \{(1206.25 + 1343.75) + (1343.75 + 1431.25) + (1431.25 + 1462.5) + (1462.5 + 1550) + (1550 + 1512.5) + (1512.5 + 1487.5) + (1487.5 + 1462.5) + (1462.5 + 1462.5) + (1462.5 + 1462.5) + (1462.5 + 1056.25)\} + (68.75 \times 131.25) + (87.5 \times 56.25)$$
$$= 44.0625 \times \{(2550) + (2775) + (2893.75) + (3012.5) + (3062.5) + (3000) + (2950) + (2906.25) + (2906.25) + (2518.75)\} + (9023.438) + (4921.875)$$

= 1,273,031 (km²)

(b) Calculate the area using a definite integral and a best fit mathematical regression



Graph of y(km) vs. x(km) from Table 2 Data

Calculation of the Definite Integral (+ Islands)

$$Area = \int_{0}^{881.25} (1204 + 1.57x - 1.82 \times 10^{-3}x^{2}) dx + (9023.438) + (4921.875)$$
$$= 1,269,412 \text{ (km}^{2})$$

(c) Show a % error between your area calculations and the accepted country's area

Expected value of the Area of the territory: 1,349,129 (km²)

1) Percent Error for Part (a)

 $\% \ error = \left| \frac{\text{derived value} - \text{expected value}}{\text{expected value}} \right| \times \ 100\% = \left| \frac{1,273,031 - 1,349,129}{1,349,129} \right| \times \ 100\% = 5.641\%$

2) Percent Error for Part (b)

$$\% \ error = \left| \frac{\text{derived value} - \text{expected value}}{\text{expected value}} \right| \times \ 100\% = \left| \frac{1,269,412 - 1,349,129}{1,349,129} \right| \times \ 100\% = 5.909\%$$

Interpretation)

According to the calculations shown in part(a) and part(b), the estimation of the area of the Northern Territory of Australia using the trapezoidal Riemann Sum is 1,273,031km², while the estimation using the definite integral and best-fit mathematical regression is 1,269,412km². Since the actual value of the Northern Territory of Australia is 1,349,219km² according to the Encyclopedia Britannica, it is clear that **the Riemann Sum method is the most accurate** as it has an estimation closer to the actual value. This fact is also evident by the percent error calculated in part(c). In part(c), it is shown that the percent error of the area derived by the trapezoidal Riemann Sum method is 5.641%, while the percent error of the area derived using the definite integral and best-fit mathematical regression is 5.909%. Although it is a very small difference, it is evident that the percent error using the trapezoidal Riemann Sum is the most accurate.